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Unidirectional drift of fronts under zero-mean force, and broken symmetries of the rate function

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The deterministic front-ratchet effect, namely, the unidirectional transport of the bistable fronts (BFs) under the additive zero-mean ac forcing, is considered within the piecewise-linear model of the bistable system. Two different mechanisms underlying the front ratchet, two cases of the broken symmetry of (i) the rate function, and/or (ii) the external zero-mean ac forcing are analyzed. Types of unidirectional motion, some versions of the "unforced" migration of BFs, are found in both cases of the travelling (initially propagating) and the static (motionless) fronts. We show that symmetry breaking in the front ratchet could produce progressive, regressive, and reversal types of the unidirectional motion of traveling BFs. By tuning the parameters of the rate function the propagation direction of BF exhibits reversal, as a function of the amplitude of the applied ac forcing. The static BFs, which stay initially at rest, can gain the dc motion discussed if the symmetry of either the rate function or the applied ac forcing is broken. The adiabatic approximation is used. To perform a rigorous analytic treatment for the arbitrary strengths of the driving force we assume that the frequency of the applied ac forcing is small, if compared to the characteristic relaxation rates in the system.

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I. INTRODUCTION

The ratchet effect, a nonzero net drift of the particles under the oscillating zero-mean force (driver), arises in a large class of asymmetric systems when driven out of thermal equilibrium [1]. Particles in a periodic potential, lacking the spatial symmetry, can drift on average in one direction even if the average of the applied forces is strictly zero. Both versions of the stochastic (noisy driver) and deterministic (regular driver) ratchets are possible and have been discussed at length in Refs. [1]. The soliton ratchet and various mechanisms underlying the "unforced" dc motion of the solitary waves in continuous spatially extended systems have been reviewed in Ref. [2]. Both underdamped and overdamped sine-Gordon "kink ratchets" have been studied in the extensive literature, analytically and by numerical simulations too (see Ref. [3]). The role of the inertial effects on the noise-supported dc drift of the kink was recently discussed in Ref. [4].

The "front-ratchet" effect, namely, the unidirectional transport of the elementary ordered structures in a bistable dissipative system, was also discussed, in both cases of stochastic and deterministic driving [5–8]. The evolution equation of the front under the external field $f(x,t)$ reads

$$u_t - u_{zz} - cu_z + R(u) = F(f;u), \quad F = \rho(u) f(z,t), \tag{1}$$

where the function $u(z,t)$ describes the steplike field of the front, $z = x - ct$ is the travelling coordinate, and the rate function $R(u)$, which characterizes the rate of the transient processes in the system, has three zeros at $u = u_1, u_2, u_3$ (say, $u_1 < u_2 < u_3$). In the case of the bistable system one has that $R'(u_{1,3}) > 0$, and $R'(u_2) < 0$, where the prime denotes the derivative. The action of the external field f on the front is described by the "response" function F, which is presented by the "multiplicative" forcing term on the right hand side of Eq. (1). The "transfer" function $\rho(u)$ in the forcing term F describes the most frequently studied case of the weak parametric (parametrically stimulated) driving [5–7]. The shape of the transfer function depends on the externally controllable parameter being under the action of the external field f. The particular case of the noisy ratchet, namely, a noise-supported drift of the fronts under "multiplicative" noise was examined in Refs. [5–7], within the cubic polynomial model of the bistable system. A crucial feature of the "parametric" ratchets is that the mean value of the forcing term F is nonzero even if the external forcing f has a zero mean. One can say that the dc motion discussed, the shift of the mean velocity of the front, comes from the different "symmetry" of the external forcing f and the response function F which describes the "actual" driving force of the front. Broadly speaking, the symmetrically oscillating force f is transformed (modified) into an asymmetrically oscillating torque F through the action of the applied field f on the externally controllable parameter of the system. As a consequence, the averages of the fields f and F differ. This produces a spurious drift, the "unforced" migration of the front. In the case of the noisy driver this conclusion may be substantiated by Novikov's theorem: the problem of the mean velocity of the disturbed front is reduced to the deterministic equation, which is characterized by the "renormalized" (modified) rate function (e.g., see in Ref. [7]). The considered ratchet effect is purely deterministic, in essence, that is, not associated with the noisy character of the driver. The mechanisms responsible for the "unforced" migration are of the same physical origin in both cases of the deterministic and stochastic driver. Clearly, the mechanism of the "parametric" ratchet breaks down if the transfer function $\rho(u)$ is a constant, i.e., in the case of the additive forcing term, $F \equiv f$ [5]. Nonetheless, the unidirectional transport of the fronts may take place even in the case of the "additive" zero-mean

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driving. A zero-mean force could give a net contribution to the front dynamics despite the additive character of the forcing term [8,9].

The "additive" front ratchet (AFR), the unidirectional transport of the bistable fronts under the additive forcing, was already considered in Refs. [5,8], by the use of the cubic polynomial approximation of the rate function. By a bistable front (BF) we mean one of "saddle-saddle" type. The considered BF separates two stable uniform states of the system, i.e., it performs the transition between the steady states u_1 and u_3. Similar to the case of the parametric ratchet, the "additive" ratchet is more efficient, and the shift of the mean velocity of the driven BF is more pronounced at high strengths of the applied forcing f At low intensities of the external field f which is tractable within the first-order approximation of the perturbation technique, the deviations of the moment velocity of the BF depend linearly on the applied forcing f Hence, weak additive driving does not produce the "spurious" drift, the unforced migration of the BF (e.g., see Refs. [5,10]).

The deterministic version of the "additive" ratchet, namely, the unidirectional transport of BFs under the periodically oscillating zero-mean forcing $f(t)$ was recently discussed in Ref. [8], within the cubic polynomial model of the system. The calculations, which have been carried out for the arbitrary strength of the applied forcing $f(t)$, showed that the "pulling" effect, the progressive dc motion of BFs, occurred if the Maxwellian construction of the rate function was not balanced. This implies that the work of the additive zero-mean driver may be converted into the accelerated dc motion of a BF if the "global symmetry" of the rate function is broken. Direct calculations showed that a significant driving effect could be achieved even in the case of an additive driver, if the applied forcing f was strong enough. Furthermore, the "pulling" of the front disappeared for the initially static BF, when the Maxwellian construction was strictly balanced.

The unforced dc motion usually originates in systems lacking some symmetry, or it comes from an asymmetrically oscillating zero-mean field $f(x,t)$ [1–3]. It is clear enough that the characteristic features of the directed transport in the AFR must be sensitive to the "symmetry" of the rate function, i.e., they should depend on the shape of the $R-u$ characteristic, similar to the case of the soliton or string ratchets [2,3]. Evidently, the "cubic polynomial" AFR is very specific, the cubic polynomial rate function satisfies some symmetry properties, discussed below. Thus, the natural question that needs to be addressed is whether the "pulling" of BFs, the progressive dc motion that occurred within the cubic polynomial model, was a common property, a generic feature of the considered AFRs. Furthermore, it is not clear enough whether or not the considered ratchet effect may take place in the "globally symmetric" case, when the Maxwellian construction is strictly balanced. Is it possible to accelerate (on average) the motionless BF, which stays initially at rest? Finally, what would be the preferred direction of the unforced migration in such a "strictly bistable" case?

Unfortunately, the analytically tractable models of front ratchets of the "flexible" symmetry are lacking. An analytic solution of the driven system with similar but differently shaped rate functions is ordinary not feasible. Thus, it seems reasonable to replace the nonlinearity by linear pieces. In the present paper a "pseudolinear" AFR that is characterized by the piecewise-linear rate function $R(u)$ is considered. The piecewise-linear approximation exhibits the most important features of the bistable system, and provides us with a model of the "flexible" symmetry. The motivation of the present study is to present a simplified picture of the unforced transport of BFs, generated by asymmetrically shaped rate functions. We extend our previous study of the ac driven BFs by a consideration of the "symmetry breaking" in AFRs that are described by differently shaped rate functions. Both the initially static and initially propagating BFs are discussed. New interesting versions of the unforced migration of BFs are found.

The unidirectional motion of BFs, induced by the additive zero-mean forcing, as far as we know, has not been studied in considerable detail as yet. In addition, the mechanisms underlying the "additive" ratchet will also work in the case of a parametric ratchet, which is described by a "multiplicative" forcing term, but not the reverse. The reverse is not necessarily true.

The paper is organized as follows. In Sec. II we discuss the model, the front solutions of the quasistatically driven BF, and the speed equation which describes the unforced migration of the ac driven BF. Section III deals with the broken symmetries of both the rate function and the external ac forcing. Two separate cases of the symmetry breaking, the characteristic features of the directed transport generated by both the asymmetric rate functions and the asymmetrically oscillating zero-mean forcing, are considered. The typical versions of the unforced migration in the considered AFR are classified. Finally, we summarize the main conclusions.

II. MODEL, APPROXIMATION, AND FRONT SOLUTIONS

The analytic treatment of the disturbed BF usually requires the use of approximate approaches. In the considered case of the AFR the perturbative techniques which describe the particular case of the weak forcing f are of limited use. The first-order approximation of the perturbation technique, as noted, leads to the vanishing result. Thus, higher order approximations of f are required to describe the unforced migration of the ac driven BF. Furthermore, the dc drift, the shift of the mean velocity of the BF, is more strongly pronounced at high strengths of the applied forcing $f(t)$ [8]. Therefore, to describe the most general case of arbitrary f's we restrict ourselves to the case of the adiabatically slow driving. We assume that the period of the ac forcing T is large if compared to the characteristic relaxation times in the system (e.g., see Refs. [8,9]). The domain of validity, a rough criterion of the adiabatic approximation, may be obtained by the use of the perturbation technique and was presented in Ref. [8]. The criterion reads $T_f \gg \tau$, where the parameter $\tau = \max\{R'^{-1}(u_1), R'^{-1}(u_3)\}$, stands for the characteristic relaxation time of the system, and the quantity T_f indicates the period of the rapidly oscillating "mode" of the forcing $f(t)$. Clearly, the periodically oscillating forcing $f(t)$ may be pre-

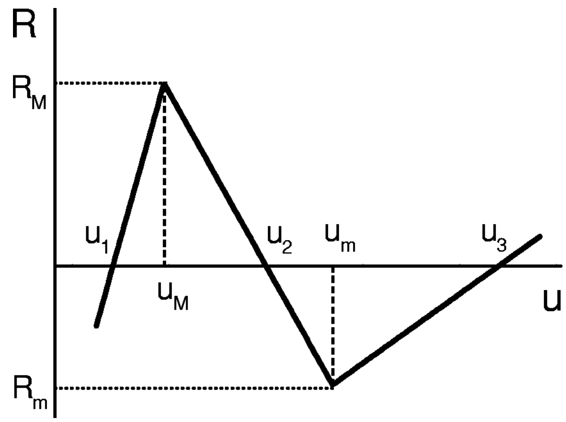

FIG. 1. The piecewise-linear rate function.

sented as a superposition of the harmonically oscillating "modes." Evidently, the discussed approximation is exact in the limit $T_f \to \infty$. The adiabatic approximation is also useful in another respect: the mechanisms underlying the considered AFR may be readily understood by considering the quasistatic limit. The response of the bistable front to the slowly oscillating forcing $f(t)$ is described by the equation

$$u_{zz}+c(t)u_z-R_F[u;f(t)]=0, \quad R_F=R(u)-f(t). \quad (2)$$

Here, by R_F we denote the modified rate function; the parameter $c(t)$, defined by the relation $c(t)\equiv c[f(t)]$, indicates the moment velocity of the driven BF, and the "flexible" rate function $R(u)$ is described by the linear pieces (see, Fig. 1)

$$R(u)=\begin{cases} \alpha_1 \ (u-u_1), & u<u_M \\ -\alpha_2 \ (u-u_2), & u_M<u<u_m \\ \alpha_3 \ (u-u_3), & u>u_m . \end{cases} \quad (3)$$

Here the free parameters of the "basic" rate function $R(u)$ are defined as follows: $u_1<u_M<u_2<u_m<u_3$, and $\alpha_i>0$, $(i=1,2,3)$. The extremes of the rate function, $R_M\equiv R(u_M)$ and $R_m\equiv R(u_m)$, are given by the expression $R_{M,m}=\alpha_2(u_2-u_{M,m})$, and the time-dependent quantities v_i, which indicate zeroes of the modified rate function R_F, are described by the relations $v_{1,3}=u_{1,3}+\alpha_{1,3}f(t)$ and $v_2=u_2-\alpha_2 f(t)$. We do not impose any additional restrictions on the parameters u_i and α_i. Thus, the shape of the rate function $R(u)$ may be easily modified through the direct variation of the adjustable parameters u_i and α_i. Clearly, in the case of the slowly oscillating (quasi-static) forcing the response of the bistable system to the applied forcing $f(t)$ may be treated as very rapid, almost "instantaneous" one (in the time scale of the characteristic period T_f of the forcing). Thus, the characteristic parameters of the disturbed front-solution, e.g., the discussed parameters $v_{1,3}$, the moment velocity $c(t)$, etc., are directly expressed through the "variable" $f(t)$. Let us turn to the front solutions of the disturbed BF.

Thanks to the piecewise-linear character of the rate function, the method of finding the front solutions is very simple. Similar to the case of the free $(f\equiv 0)$ system, both the front solution and propagation velocity of the driven BF are found by solving the system of ordinary (linear) differential equations, used in conjunction with the appropriate boundary and matching conditions (see Ref. [9]). The quasistatic field of the ac driven BF $u(z,t)$ must tend to the values $v_1(t)$ and $v_3(t)$, for $z\to\pm\infty$. Without loss of generality we assume that $u(z\to-\infty)\to v_1$, and $u(z\to+\infty)\to v_3$. This implies that the propagation velocity of the free, undisturbed BF c_0 is described by the relations $c_0>0$ if $S_M>-S_m$ and $c_0<0$ if $S_M<-S_m$, where the quantities S_M and S_m denote the areas enclosed by $R-u$ characteristic in the intervals $[u_1,u_2]$ and $[u_2,u_3]$ of the variable u respectively. More specifically, the free BF propagates in such a way that the steady state u_1 invades the state u_3, if the Maxwellian construction is "positively" disbalanced, namely, if the condition $S\equiv S_M+S_m>0$ is fulfilled. Differently, the penetrated state of the free BF is u_1 if $S<0$. For the considered case of the pseudolinear rate function (3) we get that

$$c_0 \geqslant 0 \quad \text{if} \ \ h_R \geqslant h_R^0, \quad (4a)$$

$$c_0 \leqslant 0 \quad \text{if} \ \ h_R \leqslant h_R^0, \quad (4b)$$

where

$$h_R=g_H \ h_R^0, \quad h_R^0=\sqrt{j_3/j_1}, \quad g_H=\sqrt{-S_M/S_m}. \quad (4c)$$

Here the auxiliary parameters h_R and h_R^0, defined by the relations $h_R=-R_M/R_m$ and $h_R^0=h_R(c_0=0)$, indicate the ratio of the extreme values of the rate function. The factor g_H may serve as the "balance pointer": it indicates the "disbalance" of the Maxwellian construction from the strictly balanced situation, when the equal areas rule $S_M=-S_m$ is satisfied. More specifically, the considered system is "strictly" bistable, i.e., the free BF is static (motionless) if the equality $g_H=1$ holds. Relations (4) describe the well known result: the free BF always propagates in such a way that the less stable state is the penetrated one. It will be shown below that the unforced migration of BFs does not satisfy relations (4), i.e., the given classification "scheme" breaks down. More exactly, inequalities (4a) and (4b) do not hold if one substitutes $\overline{c}$ for c_0 into Eq. (4), where by $\overline{c}$ we denote the mean velocity of the ac driven BF.

The solution of Eq. (2), the method of finding the front solution and the velocity $c(t)$, is quite similar as in the case of the free BF (see in Ref. [9]). Without giving the details of the calculations, which are straightforward albeit lengthy, we present the front solution, which satisfies the discussed boundary conditions:

$$u(z)=\begin{cases} v_1+(u_M-v_1)\exp(k_1 z), & z<0\\ v_2+(v_2-u_M)b(w)\exp(-wz)\sin(q_2 z-\Psi), & 0<z<z_m\\ v_3-(v_3-u_m)\exp[k_3(z-z_m)], & z>z_m. \end{cases} \tag{5}$$

For the sake of brevity we have introduced the abbreviation $w=c(t)/2$. The required parameters are described as follows:

$$k_{1,3}(w)=-w\pm\sqrt{w^2+\alpha_{1,3}},\quad q_2(w)=\sqrt{\alpha_2-w^2}, \tag{6a}$$

$$b(w)=\{1+1/\theta^2(w)\}^{1/2},\quad \theta(w;\delta_1)=\frac{q_2(w)}{g_1(w)}, \tag{6b}$$

$$\Psi=\begin{cases}\arctan[\theta(w)], & \theta(w)>0\\ \pi-\arctan[-\theta(w)], & \theta(w)<0,\end{cases}$$

$$z_m=q_2^{-1}\Phi(w;\alpha_1,\alpha_3). \tag{6c}$$

The auxiliary functions are given by the expressions

$$g_{1,3}=-w+\delta_{1,3}k_{1,3}(w),$$

$$\Phi(w)=\begin{cases}\arctan[\mathrm{Tg}(w)], & \mathrm{Tg}(w)>0\\ \pi-\arctan[-\mathrm{Tg}(w)], & \mathrm{Tg}(w)<0,\end{cases} \tag{7a}$$

where

$$\mathrm{Tg}(w)=f_{\mathrm{Sn}}/f_{\mathrm{Cn}},\quad f_{\mathrm{Sn}}=q_2(w)[\delta_1 k_1(w)-\delta_3 k_3(w)], \tag{7b}$$

$$\mathrm{Sn}(w)=f_{\mathrm{Sn}}/f_V,\quad f_{\mathrm{Cn}}=-[q_2^2(w)+g_1(w)g_3(w)], \tag{7c}$$

$$\mathrm{Cn}(w)=f_{\mathrm{Cn}}/f_V,\quad f_V=q_2^2(w)+g_1^2(w). \tag{7d}$$

Here the following denotations have been introduced; $\delta_i=\alpha_2/\alpha_i$ and $j_i=1+\delta_i$.

The central subject of the present consideration is the velocity of the ac driven BF. Similar to the case of the free BF (see Ref. [9]), the speed equation of the driven BF is followed from the matching conditions, by the direct substitution of the front solution (5) into the matching equations. It was shown in Ref. [9] that the propagation velocity of BF should satisfy the relation $|c|<c_P$, where the quantity $c_P\equiv 2\sqrt{-R'(u_2)}=2\sqrt{\alpha_2}$ indicates the "marginal" velocity of the "pulled" front. More accurately, the following relations hold: $c\to -c_P$ if $g_H\to 0$, and $c\to c_P$ if $g_H\to\infty$. To simplify the expressions we introduce the abbreviation $s=c(t)/c_P$. Hence, the scaled velocity of the driven BF $s(t)$ will satisfy the relation $|s(t)|<1$. Without going into detail we present the required speed equation in two completely equivalent forms,

$$H_S(s;r_1,r_3)=h_F(f), \tag{8a}$$

$$P_S(s;r_1,r_3)=p_F(f),\quad P_S(s):=1/H_S(s),$$

$$p_F(f):=1/h_F(f), \tag{8b}$$

where

$$H_S(s)\equiv\frac{Sn(s)}{\exp[-\varphi(s)]\sin\Phi(s)},$$

$$h_F(f)=h_R\frac{1-(1+p_R)f^*}{1+(1+h_R)f^*}\equiv\frac{R_M-f(t)}{-R_m+f(t)}, \tag{9a}$$

and the sign ":=" denotes a definition. Here the function $f^*=f/\Delta R$ describes the external forcing, scaled in the units of the "height" of the rate function, $\Delta R=R_M-R_m$. The parameters $r_{1,3}$ and p_R are defined as follows: $p_R=1/h_R$ and $r_i=\alpha_i/\alpha_2$. The unknown function $\varphi(s)$ in Eq. (9a) is given by the following expressions:

$$\varphi(s)=\frac{s\Phi(s)}{Q_2(s)},\quad Q_2(s)\equiv q_2/\sqrt{\alpha_2}=\sqrt{1-s^2}. \tag{9b}$$

From Eq. (9a), in the conjunction with Eqs. (6a) and (7), it follows that the quantity $H_S(s;r_1,r_3)$ in the speed equation (8a) is a function of both the velocity s and the slope parameters $r_{1,3}$. Consequently, Eq. (8a) yields by inversion that $s=s(f^*;h_R,r_1,r_3)$. Thus the moment velocity of the driven BF $s(t)=s[f^*(t);h_R,r_1,r_3]$ depends on the slope parameters $r_{1,3}$ and the "balance factor" g_H. Thus, the pseudolinear rate functions, taken at the fixed slope parameters r_1 and r_3, may be treated as "*similarly shaped*" ones. The family of similarly shaped rate functions will exhibit similar driving properties of BFs. More specifically, the characteristic features of the unforced transport generated by the rate functions with the fixed parameters r_1, r_3, and h_R will strictly coincide, if the propagation velocity and the external forcing were taken in the scaled units, defined above. This implies that the similarly shaped rate functions may be treated as equivalent, if the balance parameter g_H was taken fixed. Furthermore, the following relations hold:

$$H_S(s;r_1,r_3)=P_S(-s;r_3,r_1), \tag{10a}$$

$$h_F(f^*;h_R)=p_F(-f^*;1/h_R). \tag{10b}$$

Now, from Eqs. (9) used in conjunction with Eq. (10) immediately follows that the moment velocity of BF $s(t)$ satisfies the relation

$$s(f^*;r_1,r_3,h_R)=-s(-f^*;r_3,r_1,1/h_R). \tag{11a}$$

In addition, the auxiliary function $H_S(s)$ is positive, $H_S>0$. More exactly, the following relations hold: $H_S(s\to 1)\to\infty$, and $H_S(s\to -1)\to 0$. As a consequence, from Eq. (8a), in conjunction with Eq. (9a), follows at once that the applied

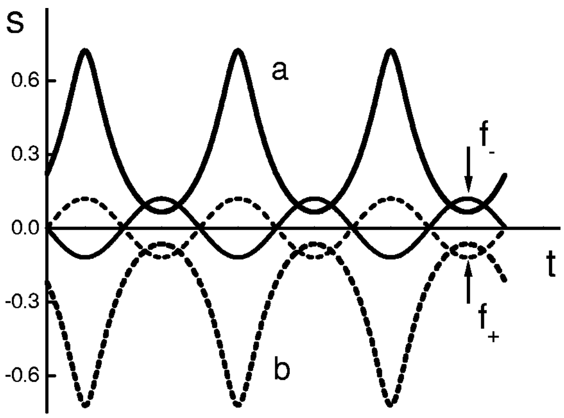

FIG. 2. The moment velocity of the bistable front driven by the harmonic forcing $f^*_{\pm}(t) = \pm f^*_0 \sin \omega t$. The parameter values are (solid line a) $f^*(t) \equiv f^*_{-}(t)$, $f_{mx} = -R_m$, $r_1 = 10$, $r_3 = 0.1$, and $h_R = 9.5$; (dashed line b) $f^*(t) \equiv f^*_{+}(t)$, $f_{mx} = R_M$, $r_1 = 0.1$, $r_3 = 10$, and $h_R = 0.105$. The rigorous equality $h_R(a) = h_R^{-1}(b)$ holds.

forcing should satisfy the condition $f(t) < f_{mx} \equiv \min\{R_M, -R_m\}$. Otherwise, using the scaled units we get that $f^*(t) < f^*_{mx} \equiv \min\{(1+p_R^{-1}),(1+h_R^{-1})\}$. It is easily to see that the presented inequalities guarantee the "global" stability of the driven BF: the middle zero point v_2 of the modified function R_F and one of the outer points, either v_1 or v_3, closely approach each other in the limit $f^* \to \pm f^*_{mx}$, and coalesce at this limit. The discussed symmetry property (11a) is illustrated in Fig. 2, where the moment velocity $s(t)$, followed from the speed equation (8a), is presented for two opposite cases of the harmonic forcing described by the relation $f^*(t) \equiv f^*_{\pm} = \pm f^*_0 \sin \omega t$. One can see that the oscillations of the velocity $s(t)$, shown by curves (a) and (b), are strictly opposite, in accordance with Eq. (11a). As a consequence, the mean velocity of the "harmonically" driven BF $\overline{s}$ will satisfy the relation

$$\overline{s}(r_1, r_3; f^*_0, h_R) = -\overline{s}(r_3, r_1; f^*_0, 1/h_R), \tag{11b}$$

where the overbar denotes the average over the period of the forcing, $T = 2\pi/\omega$. Here and in the following we use the denotation

$$\overline{A} \equiv \langle A \rangle = \frac{1}{T} \int_0^T dt A(t). \tag{12}$$

From Eq. (11a) it follows that relation (11b) holds in any case of the "symmetrically" oscillating forcing defined the relation $f(t+T/2) = -f(t)$. For this reason, when considering the directed drift of BFs that are driven by the "symmetric" ac forcing we shall restrict our consideration to the rate functions defined by the relation $r_1 > r_3$. We assume in the following that the "asymmetry factor" $\gamma_R = r_1/r_3$ satisfies the condition $\gamma_R > 1$.

III. SYMMETRY BREAKING IN FRONT RATCHETS: UNIDIRECTIONAL TRANSPORT OF AC DRIVEN FONT

Before analyzing the unforced transport of BFs, generated by the differently shaped rate functions, let us classify the relevant symmetries of both the rate function and the periodically oscillating driver. To perform the needed classification we define the rigorously symmetric functions $R(u)$ and $f(t)$ as follows: (a) $R(u_2 - \Delta u) = -R(u_2 + \Delta u)$ and (b) $f(t + T/2) = -f(t)$, where Δu is the arbitrary constant. The presented relations mean that the both discussed functions, $R(u)$ and $f(t)$, are "contrasymmetric." Roughly speaking, the considered $R-u$ and $f-t$ dependencies exhibit a "mirror image" symmetry with respect to the abscissa axis. Both the rate function $R(u)$ and the forcing $f(t)$ will be referred to as asymmetric if the relations (a) and (b) are broken. Two separate cases of the broken symmetry of the rate function $R(u)$ and/or the ac forcing (driver) $f(t)$, two different types of the symmetry breaking in the considered AFR could be identified: (A) $R(u_2 - \Delta u) \neq -R(u_2 + \Delta u)$, and/or (B) $f(t + T/2) \neq -f(t)$. Both lead, as shown below, to a "rectification" of the moment velocity of the ac driven BF and, as a consequence, to the front-ratchet effect. We notice that the asymmetric driver, defined by relation (B), is frequently used as the basic "*impetus*" in the Brownian motors [1]. Using the discussed relations we arrive at the following combinations of the symmetry breaking in AFRs: (A-b)—the rate function is asymmetric, the driver is symmetric; (B-a)—the driver is asymmetric, the rate function is symmetric; (A-B)—both, the rate function and the driver, are asymmetric. In the present report we shall deal mainly with the case (A-b). Primary attention will be given to the speed "rectification" generated by the asymmetrically shaped rate functions. It is easy to see that the cubic polynomial AFR, discussed in Ref. [8], may be typified as the case (A-b). Nevertheless, the cubic polynomial rate function is, to some degree, symmetric. Let us specify the symmetry properties of the rate functions more accurately. It is obvious that the structural asymmetry of the considered (spatially homogeneous) system is masked in the rate function.

The rigorously symmetric rate function defined by relation (a) implies that the considered system is "strictly bistable." The equal areas rule $S_M = -S_m$ holds; hence, the free BF is always static in such a "locally" symmetric case. Clearly, the equal areas rule may be fulfilled too in the other case of the asymmetric rate functions defined by relation (A). Generally speaking, the rate functions, which satisfy the equal areas condition, may be typified as follows: (1) $R_s(u_2 - \Delta u) = -R_s(u_2 + \Delta u)$ and (2) $R_a(u_2 - \Delta u) \neq -R_a(u_2 + \Delta u)$. Hence, the two separate classes of the similarly shaped rate functions may be identified: (I) $\{R_S\} := R_s(u) + C$—the *symmetrically shaped* (symmetrical) ones, and (II) $\{R_A\} := R_a(u) + C$—the *nonsymmetrically shaped* (asymmetrical) those, where C denotes the free constant. Evidently, the criterion of the "broken symmetry" (A) may fulfilled in either case of the *symmetrical* (R_S) or *asymmetrical* (R_A) rate function. Nevertheless, the peculiarities of the unidirectional motion of BFs, i.e., the characteristic types of the unforced migration, generated in either case of the symmetrically or the asymmetrically shaped rate function, are rather different. For instance, the rate functions described by the cubic polynomials are "symmetrically shaped" (symmetrical) ones. As a consequence, the "pulling" of the fronts, the particular case of the accelerated dc motion of the traveling (initially propagating) BFs occurred in that case [8]. In what follows we shall deal with the most general case of the asymmetrical rate functions, which satisfy the relation (II). The new versions of the directed motion of BFs will be found with the asymmetrical rate functions.

Referring to the particular case of the piecewise-linear approximation we write $R_A(u) \equiv R(u;\gamma_R \neq 1)$ and $R_S(u) \equiv R(u;\gamma_R = 1)$. Thus, the symmetry properties of the pseudolinear rate functions are very transparent; they are "governed" by the asymmetry factor γ_R. We notice that the rigorously symmetric rate function (a), which describes the particular, very specific case of the symmetrical rate functions, is defined by the relations $h_R = 1$ and $\gamma_R = 1$.

In closing the discussion of the symmetry properties of the considered AFR we summarize that two separate cases of the broken symmetry of (i) the rate function, and (ii) the external driver could be identified. In addition, two different classes of the asymmetric rate functions may be typified: (I) symmetrical (symmetrically shaped) ones, and (II) asymmetrical (asymmetrically shaped) those. Let us turn to the unforced transport of the ac driven BFs, generated by the asymmetrically shaped rate functions. As assumed, we shall deal with the similarly shaped rate functions that satisfy the relation $\gamma_R > 1$.

A. Unidirectional transport generated by asymmetrically shaped rate functions

Speed equation (8a) is transcendental, the velocity $s[r_1, r_3; f^*(t)]$ is not expressible as function of the parameters r_1, r_3, g_H and the forcing term $f^*(t)$. Thus, the speed equation was examined numerically, the numerical simulations have been used to describe the unforced migration of BFs. As a preliminary let us discuss the typical versions of the directed drift stimulated by the periodic square-pulse forcing,

$$f^*(t) = \begin{cases} f_0^*, & nT < t < (n+1/2)T \\ -f_0^*, & (n+1/2)T < t < (n+1)T, \end{cases}$$

$$n = 0, \pm 1, \pm 2, \ldots . \tag{13}$$

The $\overline{s} - f_0^*$ characteristics of the ac driven BF, following from speed equation (8a), are shown in Fig. 3, for the different "balance" parameters g_H. The strengths of the applied forcing and the slope parameters were taken in accordance with the relations $0 \leqslant f_0^* < f_{mx}^*$ and $\gamma_R > 1$, discussed above. The $\overline{s} - f_0^*$ characteristics shown by dashed lines a and b describe the "standard" pulling effect, the progressive dc motion of the traveling ($s_0 \neq 0$) BF, previously discussed by use of the cubic polynomial rate function [8]. The progressive dc motion implies that the average velocity of the traveling BF was increased due to the action of the external zero-mean forcing on the front. The discussed type of directed drift takes place even in the case of the symmetrically shaped rate functions, when the asymmetry factor γ_R equals unity. Nevertheless, the rigorously symmetric rate function, which describes the particular case of the initially static BF, does not exhibit the dc motion discussed. Quite similar to the case of the cubic polynomial model, the static BF, which stays initially at rest, cannot gain the directed motion discussed if the relations $\gamma_R = 1$ and $h_R = 1$ are fulfilled. Different types of the directed motion, the new versions of the unforced migration, are shown by solid curves 1, 2, and 3. They could be typified as follows: the progressive dc motion of the static BF (curve 1), the reversal migration of the traveling BF (curve 2), the stopping of the traveling BF (curve 3). We stress that the considered versions take place only in the case of the asymmetrically shaped rate functions, defined above. The symmetrical rate functions do not exhibit the discussed types of the unforced migration, very similar to the case of the cubic polynomial model. More specifically, the curves (1, 2, and 3) shown in the figure become more and more flattened and approach the segment [0,1/2] of the straight line described by the equation $\overline{s} = 0$, if the asymmetry factor γ_R tends to unity. The $\overline{s} - f_0^*$ characteristics shown by curves 1 and 3 demonstrate that the two check points, two "critical" values of the balance factor g_H could be identified, namely, $g_H = 1$ and $g_H = 1/h_R^0$. Both of these possess a similarity, namely, they are related to the motionless BF. The first deals with the initially static BF, whereas the second stands for the other limiting case of the "motionless" BF that is driven at the maximal strength of the forcing, $f_0^* = f_{mx}^*$. The latter implies that the average velocity of BF, which at the beginning was propagating at some fixed non-zero velocity s_0, was reduced to zero, i.e., the following relations $s_0 \neq 0$ and $\overline{s}(f_{mx}^*) = 0$ were satisfied. The novel types of directed motion satisfy the following relations: (1) $g_H = 1$—the accelerated dc motion of the static BF (curve 1); (2) $1 > g_H > (h_R^0)^{-1}$—the reversal migration of the travelling BF (curve 2); (3) $g_H = (h_R^0)^{-1}$—the decelerated drift, followed by the stopping effect (curve 3). One can see that the character of the $\overline{s} - f_0^*$ dependencies, shown by curves 3 and b, is rather different. The interval of the intermediate values g_H that separate the curves 3 and b depends on the asymmetry factor γ_R and is relatively small. In addition, the "intermediate" $\overline{s} - f_0^*$ characteristics are nonmonotonic functions of the amplitude f_0^* (see dotted curve c in Fig. 3). It should be noted

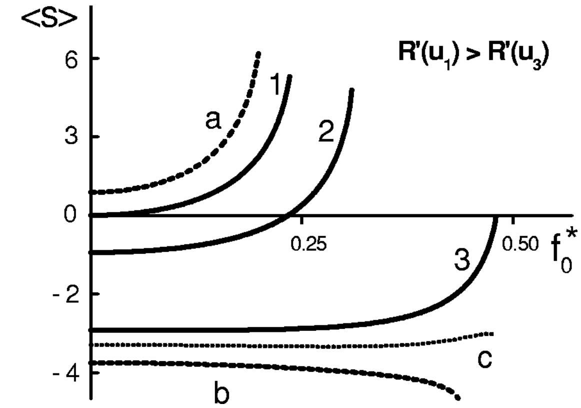

FIG. 3. The characteristic types of the unidirectional motion induced by the periodic square-pulse forcing. The slope parameters are $r_1 = 10$ and $r_3 = 0.1$. Other parameter values are (dashed line a) $g_H = 1.25$ ($h_R > h_R^0$); (solid line 1) $g_H = 1$ ($h_R = h_R^0$); (solid line 2) $g_H = 0.7$ ($h_R^0 > h_R > 1$); (solid line 3) $g_H \approx 0.32$ ($h_R = 1$); (dashed line b) $g_H = 0.25$ ($h_R < 1$); (dotted line c) $g_H = 0.29$ ($h_R < 1$). The mean velocity is presented in arbitrary units, $\overline{s_0}(a) = 1$.

that the novel types of the directed motion take place in any case of the symmetrically oscillating forcing. This conclusion may be substantiated in the following way.

First, the obvious relations hold: $f_{mx}=-R_m$ if $h_R>1$, and $f_{mx}=R_M$ if $h_R<1$. As assumed, the asymmetry factor γ_R satisfies the relation $\gamma_R>1$, thus, we get that $h_R^0>1$, in accordance with Eq. (4c). Now is easily to conclude that following relations are fulfilled: (i) $s_{mx}^-=1$ and $s_{mx}^+>-1$ if $h_R>1$; and (ii) $s_{mx}^-<1$ and $s_{mx}^+=-1$ if $h_R<1$, where $s_{mx}^\pm\equiv s(\pm f_{mx}^*)$. Hence, for the particular case of the initially static BF we obtain that $\overline{s_0}=0$, and $\overline{s_{mx}}>0$, where the quantity $\overline{s_0}\equiv\overline{s}(f_0^*=0)$ indicates the initial velocity of the BF, and by $\overline{s_{mx}}$ we denote the mean velocity taken at the maximal driving force, $f_0^*=f_{mx}^*$. It is easy to see that the obtained relations $\overline{s_0}=0$ and $\overline{s_{mx}}>0$ are in a qualitative agreement with the result shown by curve 1 in Fig. 3. Considering the other case of the "motionless" BF, shown by curve 3, we take that $g_H=1/h_R^0$. Thus, we get that $s_0<0$, in accordance with Eq. (4). Further, the relations $s(g_H=0)=-1$ and $s(g_H=\infty)=1$ hold, in accordance with the findings of Ref. [9]. Now follows at once that $\overline{s_0}(h_R=1)<0$ and $\overline{s_{mx}}(h_R=1)=0$. Whence, the mean velocity of the ac driven BF is a decreased function of f_0^*, and the "stopping" effect was occurred. At last, the "reversal" type of the directed migration (curve (2)) may be substantiated in a similar manner. It is easily to show that the reversion of the unforced migration will take place in any case of the asymmetrical rate function, even if the asymmetry factor γ_R is close to unity. To avoid the confusion we denote the balance parameter g_H of the "reversal" BF by r_H. Thus, we rewrite the above discussed relation (2) as follows, $1<r_H<(h_R^0)^{-1}$, or differently, $h_R^0<h_R<1$. Now, owing to the relation $\gamma_R>1$ we get that $f_{mx}=-R_m$. Further, the obvious relations $\overline{s_0}(r_H)<\overline{s_0}(g_H=1)=0$ and $\overline{s_{mx}}(r_H)>\overline{s_{mx}}(g_H=1/h_R^0)=0$ hold. Whence it follows at once that the inequalities $\overline{s_0}(r_H)<0$ and $\overline{s_{mx}}(r_H)>0$ are satisfied, i.e., the initial and "final" velocities of the unforced motion are opposite in sign. Consequently, the reversal type of the directed motion occurred.

We summarize by noting that the new types of the unidirectional transport of BFs will take place in any case of the symmetric ac forcing, if the rate function is asymmetrically shaped, i.e., if the relation $\gamma_R\neq 1$ holds. The discussed versions of the directed motion are somewhat different from those given in the case the cubic polynomial rate function [8]. Namely, the "stopping" effect and the reversal type of the directed migration of the traveling BFs, as well as the progressive dc motion of the initially static BF, were absent in the cubic polynomial case. In particular, the possibility of the reversal migration demonstrates that the driven system could arrive at a nontrivial behavior. The "unforced" BF propagates (on average) toward the most stable state of the system. Thus, the usual "scenario" of the transition, "from the most stable toward the less stable state" was broken down. We emphasize that the discussed "anomalies," including the stopping effect and the progressive dc motion of the static BFs, vanish at the limit $\gamma_R\to 1$. The "strange" $\overline{s}-f_0^*$ characteristics, as already noted, approach the dependence $\overline{s}=0$ when the asymmetry factor γ_R tends to unity. It is

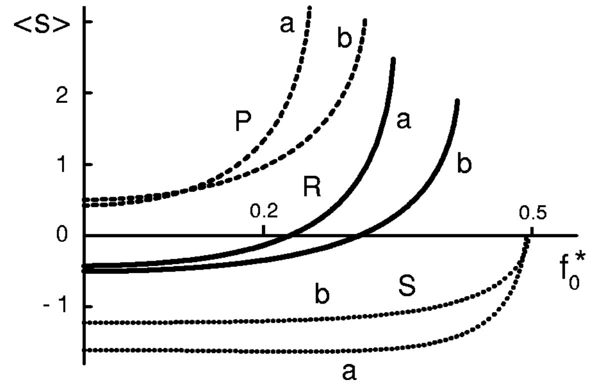

FIG. 4. The mean velocity (in arbitrary units) of the front that is driven by the harmonic forcing. The labels P, R, and S denote the progressive (P) and reversal (R) types of dc motion, and the "stopping" (S) effect, respectively. The balance factors are (dashed lines P) $g_H=1.25$; (solid lines R) $g_H=0.8$; (dotted lines S) $g_H\equiv 1/h_R^0\approx 0.43(a),0.58(b)$. The slope parameters of the rate function were taken as $r_1=1$, and (curves a) $\gamma_R=10$; (curves b) $\gamma_R=5$.

interesting to note that the possibility of the "reversal" effects in the "complex" Brownian ratchets, namely, in the "overdamped" Josephson junction device and in the noisy Brownian motor, which is characterized by the space-dependent friction coefficient, was recently discussed in Ref. [11].

We already noted that the characteristic features of the unforced migration are not sensitive to the "profile" of the symmetric ac forcing. Similarly as in the case of the "pulled" BFs [8], the shape of the periodic function $f(t)$ influences the "size" of the driving effect but does not modify the character of the $\overline{s}-f_0^*$ dependence. The $\overline{s}-f_0^*$ characteristics taken for the frequently studied case of the harmonic forcing $f^*(t)=f_0^*\sin(\omega t)$ are shown in Fig. 4, where the labels P, R, and S denote the progressive and reversal types of the directed drift, and the stopping effect, respectively. The influence of the asymmetry factor γ_R on the "size" of the driving effect is demonstrated by curves a and b, which satisfy relation $\gamma_R(a)>\gamma_R(b)$. The presented characteristics evidently show that the unforced transport is more "efficient" if the asymmetry of the rate function is more pronounced. Namely, the curves a, which correspond to the greater asymmetry parameter γ_R are more rapid, the acceleration factor $\sigma(f_0^*)=\overline{s}(f_0^*)/s_0$ taken at the fixed amplitude of the forcing f_0^* increases with γ_R, in every case labeled by P, R, and S. Curves b that correspond to the lesser asymmetry factor γ_R are more flattened. As a consequence, the reversion of the directed migration of the BF (see curves R) is achieved at a larger amplitude of the forcing f_{0r}^* if the asymmetry factor γ_R was decreased. More exactly, the following relation holds: $f_{0r}\to R_M\to -R_m$ if $\gamma_R\to 1$, whence follows that $f_{0r}^*\to f_{mx}^*=1/2$ if $\gamma_R\to 1$. Finally, the "size" of the driving effect is large enough; a ten-fold acceleration of BF may be achieved if the initial velocity of the front s_0 was small enough. The "effectiveness" of the AFR is demonstrated in Fig. 5 where the $\sigma-f_0^*$ characteristics, taken for the particular case of the progressive dc motion of the travelling BFs, are presented. As earlier, the acceleration factor

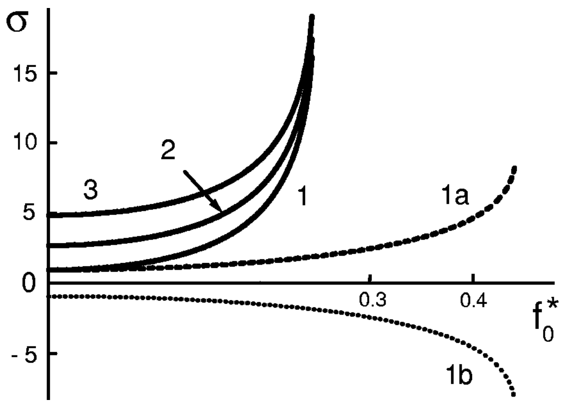

FIG. 5. The "size" of the driving effect. The parameter values are (solid lines 1, 2, 3) $h_R=3$, $r_3=0.05$, $\gamma_R=10(1),5(2),2(3)$; (dashed line $1a$) $h_R\approx 1.3$, $r_3=2$, $\gamma_R=10$; (dotted line $1b$) $h_R(1a)=h_R^{-1}(1b)\approx 1.27$, $r_3=20$, $\gamma_R=0.1$. Curves 1 and $1a$ show the dependence of the acceleration factor versus the slope coefficient α_2; the slope coefficients were taken as $\alpha_2(1)=40\alpha_2(1a)$, and $\alpha_1(1)=\alpha_1(1a)$, and $\alpha_3(1)=\alpha_3(1a)$. Curves $1a$ and $1b$ illustrate relation (11b); the parameter values satisfy the relations $h_R(1a)=h_R^{-1}(1b)$ and $\gamma_R(1a)=\gamma_R^{-1}(1b)$.

$\sigma(f_0^*)$ increases with γ_R (see curves 1, 2, 3). The maximal acceleration factor $\sigma_{mx}=\sigma(f_{mx}^*)$ is of the order of 10. Furthermore, the acceleration factor σ increases with α_2; the shift of the mean velocity is more pronounced if the middle slope coefficient of the rate function α_2 is large (see curves 1 and $1a$ in Fig. 5). The characteristic parameters of $\sigma-f_0^*$ dependencies shown by curves 1 and $1a$ were taken as follows: $\alpha_2(1)=40\alpha_2(1a)$, $\gamma_R(1)=\gamma_R(1a)$, and $s_0(1)=s_0(1a)$. Thus, the asymmetry factors and the initial velocities of BFs strictly coincide in both cases discussed. Nevertheless, the acceleration factors are rather different, curve $1a$ is more flattened. That is, one has that $\sigma_{mx}(1)/\sigma_{mx}(1a)\cong 2$, from which follows at once that $\overline{c}_{mx}(1)/\overline{c}_{mx}(1a)\cong 12$, in accordance with the discussed relation $c=2\sqrt{\alpha_2}s$. Referring to symmetry relation (11b) we remind that the characteristic features of the unforced migration are quite similar in both "contrasymmetric" cases defined by the relations $\gamma_R>1$ and $\gamma_R<1$. The unforced transport generated by the contrasymmetrically shaped rate functions $R(\gamma_R,h_R)$ and $R(1/\gamma_R,1/h_R)$ satisfies relations (11a) and (11b), discussed above. This property is illustrated by curves $1a$ and $1b$ that satisfy the relations $\gamma_R(1a)=1/\gamma_R(1b)=10$, and $h_R(1a)=1/h_R(1b)\approx 1.27$. The $\sigma-f_0^*$ characteristics shown by curves $1a$ and $1b$ are rigorously symmetric, namely, the relation $\sigma(\gamma_R,h_R;f_0^*)=-\sigma(1/\gamma_R,1/h_R;f_0^*)$ holds, in accordance with Eq. (11b).

There is little sense in discussing the peculiarities of the directed motion stimulated by the differently shaped forcing functions $f(t)$. Our calculations, carried out by use of the harmonic, biharmonic, and square-pulse functions $f(t)$, showed that the considered ratchet effect, the shift of the mean velocity of the BF is more pronounced with the symmetrically oscillating functions $f(t)$ that are characterized by the flattened "profiles" in the vicinity of the extremes $f=\pm f_0$.

We summarize by noting that the characteristic features of the directed motion of BFs that are driven by the symmetric ac forcing are rather different in the cases of the symmetrically and asymmetrically shaped rate functions. The asymmetrically shaped rate function exhibits the considered front-ratchet effect, no matter whether or not the Maxwellian construction of the rate function was satisfied. The progressive, regressive, and reversal types of the unforced migration of the traveling (initially propagating) BFs are generated with the asymmetrical rate functions. The governing parameters that significantly influence the size of the driving effect are as follows: the asymmetry factor $\gamma_R\equiv r_1/r_3$, the middle slope coefficient of the rate function α_2, and the balance factor g_H, which indicates the deviation of the Maxwellian construction from the strictly balanced situation. The maximal driving effect, the maximal the acceleration factor $\sigma_{mx}=\overline{s}_{mx}/s_0$ is achieved at the maximal amplitude of the ac forcing, $f_{mx}=\Delta R f_{mx}^*$, which linearly increases with ΔR. Thus, the shift of the mean velocity in nonscaled units, $\Delta\overline{c}(f_0)=\overline{c}(f_0)-c_0$, decreases with the increased ΔR, if the amplitude f_0 of the ac forcing was kept fixed. Finally, the rigorously symmetric rate function, defined by the relation $R(u_2-\Delta u)=-R(u_2+\Delta u)$, does not exhibit the considered ratchet effect. The motionless BF, which stays initially at rest, could gain the unidirectional motion discussed if the slope coefficients taken at the outer zero points of the rate function, $R'(u_1)$ and $R'(u_3)$, were different. The unforced migration of the motionless BF is always directed toward the domain of that stable state u_i, which is characterized by the lesser slope coefficient $R'(u_i)$; here $i=1,3$.

B. Unidirectional motion induced by asymmetrically oscillating forcing

Let us touch briefly on the unidirectional transport stimulated by the asymmetric ac forcing. As noted, the considered AFR does not work if both the rate function and the driver are rigorously symmetric. Thus, we confine ourselves to the particular case of the AFR previously labeled by (B-a). We assume that the driver is asymmetric, whereas the rate function is rigorously symmetric, namely, the relations $f(t)\neq -f(t+T/2)$, $\gamma_R=1$, and $g_H=1$ hold. Thus, the particular case of the initially static BF will be considered. The periodic ac forcing $f(t)$ may be presented as a superposition of the harmonic functions. For simplicity's sake we shall focus on the particular case of the biharmonic forcing described by the expression

$$f(t)=f_{01}\sin(\omega_1 t+\varphi_0)+f_{02}\sin(\omega_2 t+\varphi_0+\Delta\varphi). \quad (14)$$

In the considered case of the deterministic AFR the period of the ac forcing T should be a finite, uniquely defined parameter. Hence, the frequencies ω_1 and ω_2 are commensurate, and the ratio $\beta_\omega=\omega_2/\omega_1$ is the positive rational number. Different from the case of the soliton ratchets [12], the initial phase φ_0 may be chosen to be arbitrary. Indeed, the considered system is essentially dissipative (extremely overdamped), thus the "memory" is completely lost, i.e., the biharmonic forcing may be treated as adiabatically slow if the relation $T_f\gg\tau$ is fulfilled. As previously, here by τ we denote the characteristic relaxation time of the system, and the parameter $T_f = \min\{\omega_1^{-1},\omega_2^{-1}\}$ indicates the period of the rapidly oscillating "mode" of the forcing. In the considered case of the asymmetrically shaped function $f(t)$ the maximal (f_M) and the minimal ($|f_m|$) values ("amplitudes") of the forcing are different. The "degree" of the asymmetry of the biharmonic forcing may be evaluated by the asymmetry factor δ_F defined by the relation $\delta_F = -\overline{f_A}/f_N$, where by $\overline{f_A}$ we denote the average $\overline{f_A} = 0.5(f_M + f_m)$, and the "normalizing" factor f_N is given by the expression $f_N = 0.5(f_M + |f_m|)$. The biharmonic forcing will be referred to as the "positively" asymmetric one if the parameter δ_F is positive, i.e., when the inequality $|f_m| > f_M$ holds. Differently, the inequality $\delta_F < 0$ stands for the opposite case of the "negative" asymmetry. In the case of the rigorously symmetric (e.g., harmonic) forcing one has that $\delta_F = 0$. The shape of the periodic function (14) depends on the adjustable parameters ω_1, β_ω, f_{01}, and $\beta_f \equiv f_{02}/f_{01}$, and the relative phase $\Delta\varphi$. Clearly, the considered function $f(t)$ is "almost symmetric," namely, the relation $\delta_F \ll 1$ holds if either the amplitudes f_{01} and f_{02} or both frequencies ω_1 and ω_2 significantly differ from each other in magnitude. The asymmetry of the biharmonic forcing will be distinctly pronounced if the amplitudes f_{01} and f_{02}, and both frequencies ω_1 and ω_2 are of the same order in magnitude, i.e., when the relations $\beta_f \cong 1$ and $\beta_\omega \cong 1$ hold. Taking the parameters β_f, β_ω, and $\Delta\varphi$ to be fixed, we arrive at the family of "similarly shaped" functions $f(t;f_{01})$. The strength of the forcing, the "amplitude" of the similarly shaped function $f(t;f_{01})$ may be easily modified through the variation of the parameter f_{01}. Here we shall deal with the similarly shaped functions $f(t)$. We stress that the ratio $\beta_f = f_{02}/f_{01}$ is kept fixed when the "amplitude" of the asymmetrically oscillating forcing $f(t;f_{01})$ is varied. Finally, the condition $f_{mx} < \min\{-f_m, f_M\}$, which guarantees the "global" stability of the driven BF, now reads $f_{01} \leqslant (1+\beta_f)^{-1} f_{mx}$.

The $\overline{s} - f^*_{01}$ characteristics of the initially motionless BF that is driven by the asymmetric ac forcing (14) are shown in Fig. 6. The required parameters were taken as follows, $r_1 = r_3 = 1$, $\beta_\omega = 2$, and $\beta_f = 1$. The asymmetry factor δ_F was changed by tuning the relative phase $\Delta\varphi$ in accordance with the relation $\Delta\varphi = n\pi/4$, where $n = 1,2,3,\ldots$. We note that the biharmonic forcing is rigorously symmetric, i.e., the equalities $\delta_F = 0$ and $f(t+T/2) = -f(t)$ hold if one assumes that $n = 0,4,8,\ldots$ Obviously, the unforced transport of BF disappears in such a rigorously symmetric case. Differently, the $\overline{s} - f^*_{01}$ characteristics taken in the other case of the asymmetric forcing ($\delta_F \neq 0$) show that a dc motion of the initially static BFs occurred (see curves a, b, and a', b' in Fig. 6). More specifically, the following relations $\overline{s} > 0$ if $\delta_F > 0$, and $\overline{s} < 0$ if $\delta_F < 0$ hold. Thus, the unforced migration of the motionless BF is directed toward the stable state u_3 if the biharmonic forcing is "positively" asymmetric ($\delta_F > 0$), and, differently, the penetrated state of the BF is u_1 in the opposite case of the "negative" asymmetry ($\delta_F < 0$). This implies that the directed drift of the BF that was driven by the biharmonic forcing satisfied the modified Maxwellian rule, given by the following relations: $\overline{s} > 0$ if $S^* > 0$, and $\overline{s} < 0$ if $S^* < 0$. Similarly to what went previously, here by S^* we denote the total area enclosed by the modified rate function $R^*(u) \equiv R(u) + \overline{f_A}$.

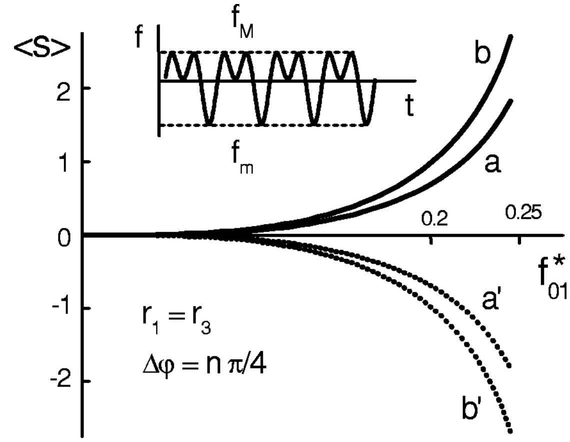

FIG. 6. The mean velocity of the initially static BF vs the "amplitude" of the biharmonic forcing. The parameters are $r_1 = r_3 = 1$, $\beta_\omega = 2$, $\beta_f = 1$, and $\Delta\varphi = n\pi/4$, where n is integer, and (solid line a, $n = 1,3$) $\delta_F \approx 0.32$; (solid line b, $n = 2$) $\delta_F \approx 0.78$; (dotted line a'; $n = 5,7$) $\delta_F \approx -0.24$; (dotted line b'; $n = 6$) $\delta_F \approx -0.44$. Other parameter values are $\gamma_F(a) = \gamma_F^{-1}(a') \approx 1.32$, $\overline{f_A}(a) = -\overline{f_A}(a') \approx 0.24$ and $\overline{f_A}(b) = -\overline{f_A}(b') \approx 0.44$, $\gamma_F(b) = \gamma_F^{-1}(b') \approx 1.78$. The inset shows the biharmonic function (14) ($n = 2$; $\Delta\varphi = \pi/2$).

The "size" of the diving effect, i.e., the steepness of the $\overline{s} - f^*_{01}$ characteristics depends on both the asymmetry factor $|\delta_F|$ and the outer slope parameters $r_{1,3}$. The acceleration factor $\delta\overline{s}/\delta f^*_{01}$ increases with $|\delta_F|$ (e.g., see curves a and b in Fig. 6). The $\overline{s} - f^*_{01}$, dependencies, taken at the same asymmetry parameter δ_F, strictly coincide, in spite of the fact that the shapes of the forcing functions $f(t)$ are slightly different [see curve a, ($n = 1,3$) and curve a', ($n = 5,7$) in Fig. 6]. Furthermore, the relation $\overline{s}(\delta_F; f^*_{01}) = -\overline{s}(-\delta_F; f^*_{01})$ holds: curves a and a', as well as those labeled by b and b' are rigorously symmetric. The discussed relation is illustrated by Fig. 7, where the maximal drift velocity of the front, $\overline{s_{mx}} \equiv \overline{s}(f_{mx})$, is plotted versus the relative phase $\Delta\varphi$, for the different slope parameters $r_{1,3}$ and the arbitrary δ_F's. One can see that the acceleration factor $\delta\overline{s}/\delta f^*_{01}$ increases if the slope parameters r_1 and r_3 decrease. This means that the considered driving effect is enhanced if either the middle slope coefficient α_2 was increased or the both outer slope coefficients α_1 and α_3 were decreased, in any case of the asymmetric forcing [Eq. (14)]. Both, the shape of the biharmonic function $f(t)$ and the asymmetry factor δ_F, as noted, are modified through the variation of the relative phase $\Delta\varphi$. The asymmetry factor δ_F periodically oscillates with $\Delta\varphi$ (see the dashed curve in Fig. 7). As a consequence, the size of the effect, the shift of the mean velocity of BF is the periodic function of $\Delta\varphi$ (see curves a, b, and c). Finally, the considered driving effect is large enough if the outer slope coefficients of the rate function $\alpha_{1,3}$ are relatively small. For instance, taking $\alpha_{1,3} \cong 0.1$ and $\alpha_2 \cong 10$ we get that $\overline{s_{mx}} \cong 0.1$. Thus the maximal velocity of the directed migration in nonscaled units $\overline{c_{mx}}$ is given by the relation $\overline{c_{mx}} \cong 0.1 c_P$, where the quantity $c_P \equiv 2\sqrt{\alpha_2}$ indicates the propa-

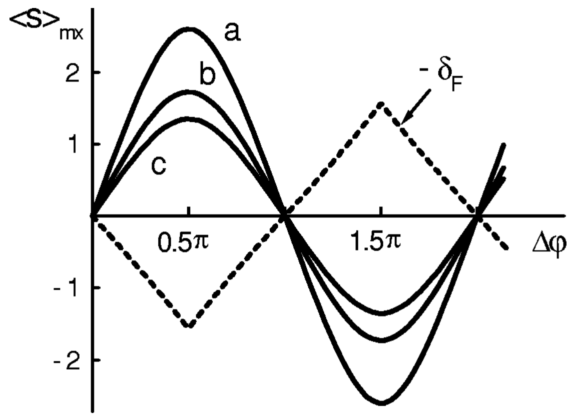

FIG. 7. The maximal drift velocity $\overline{s_{mx}}$ (in arbitrary units) of the motionless BF vs the relative phase of the biharmonic forcing $\Delta\varphi$. The parameters are $\beta_\omega = 2$ and $\beta_f = 1$. The parameters of the rate function: $\gamma_R = 1$, and $r_1 \equiv r_3 = 0.1(a), 1(b)$, and $10(c)$. The dashed line shows the dependence of the asymmetry factor δ_F vs the relative phase $\Delta\varphi$.

gation velocity of the ''pushed'' BF, which always propagates at the greatest, ''marginal'' velocity. Thus, the shift of the mean velocity increases with α_2, very similarly as in the case of the symmetrically oscillating forcing, discussed above.

We summarize by noting that the unforced transport of the BF may occur even in the case of the rigorously symmetric rate function, if the applied zero-mean ac forcing $f(t)$ is asymmetric, i.e., when the relation $f(t) \neq -f(t+T/2)$ holds. In the particular case of the biharmonic forcing, which asymmetrically oscillates with time, the progressive, accelerated dc motion of the initially static BF takes place. The mean velocity of the considered BF depends on both the relative phase of the biharmonic forcing $\Delta\varphi$ and the slope coefficients of the rate function. Both the increase of the middle slope coefficient $R'(u_2)$ and the decrease of the outer slope coefficients $R'(u_1)$ and $R'(u_3)$ enhance the driving effect discussed. The preferred direction of the directed motion of the motionless BF is described by the modified Maxwellian rule.

IV. CONCLUSIONS

The unidirectional transport of the bistable fronts under the action of the additive zero-mean ac forcing was considered within the piecewise linear approximation of the rate function. The bistable system under the periodically oscillating zero-mean forcing may serve as a deterministic front ratchet, irrespective of the symmetry properties of the rate function. Two separate classes of the deterministic ratchets based on the broken symmetry of (i) the rate function $R(u)$, and/or (ii) the external ac forcing $f(t)$ have been analyzed. It was shown that the work of the periodic zero-mean forcing might be converted into a direct motion of the bistable front, in both cases of the symmetrically and asymmetrically oscillating forcing. The progressive (accelerated), regressive (decelerated), and reversal dc motions of the travelling (initially propagating) BFs that are driven by the symmetrically oscillating ac forcing were found with asymmetrically shaped rate functions. The initially static BF, if influenced by the periodic zero-mean forcing, can gain the dc motion discussed if either the rate function or the applied ac forcing is asymmetric, namely, if one of the following relations, $R(u_2-\Delta u) \neq R(u_2+\Delta u)$ and $f(t) \neq -f(t+T/2)$, was satisfied. The governing parameters that influence the ''size'' of the ratchet effect are as follows: the strength of the applied ac forcing, both the asymmetry factor and the middle slope coefficient of the rate function, and the balance factor, which indicates the disbalance of the Maxwellian construction.

The quasistatic approximation left out the question about the dependence of the characteristics of the ratchet versus the frequency of the applied ac forcing. This problem, as well as the other case of the noisy AFR, is analytically tractable by the use of perturbative techniques, and requires further attention.

[1] P. Reimann, Phys. Rep. **361**, 57 (2002); D. Abbott, Chaos **11**, 526 (2001); A. M. Jayannavar, e-print, cond-mat/0107079; F Julicher, A Adjari, and J Prost, Rev. Mod. Phys. **69**, 1269 (1997); S. Denisov, S. Flach, A. A. Ovchinnikov, O. Yavtushenko, and Y. Zolotariuk, Phys. Rev. E **66**, 041104 (2002); M. Borromeo, G. Constantini, and F. Marchesoni, *ibid.* **65**, 041110 (2002).

[2] M. Salerno and N. R. Quintero, Phys. Rev. E **65**, 025602 (2002).

[3] A. V. Savin, G. P. Tsironis, and A. V. Zolotryuk, Phys. Rev. E **56**, 2457 (1997); N. R. Quintero and A. Sanchez, Eur. Phys. J. B **6**, 133 (1998); S. Flach, Y. Zolotaryuk, A. E. Miroshnichenko, and M. V. Fistul, Phys. Rev. Lett. **88**, 184101 (2002); G. Costantini, F. Marchesoni, and M. Borromeo, Phys. Rev. E **65**, 051103 (2002).

[4] J. M. Sancho and A Sanchez, Eur. Phys. J. B **16**, 127 (2000).

[5] A. S. Mikhailov, L. Schimansky-Geier, and W. Ebeling, Phys. Lett. A **98**, 453 (1983); L. Schimansky-Geier and Ch. Zulicke, Z. Phys. B: Condens. Matter **82**, 157 (1991).

[6] J. Armero, J. M. Sancho, J. Casademunt, A. M. Lacasta, L. Ramirez-Piscina, and F. Sagues, Phys. Rev. Lett. **76**, 3045 (1996); J. Armero, J. Casademunt, L. Ramirez-Piscina, and J. M. Sancho, Phys. Rev. E **58**, 5494 (1998); M. A. Santos and J. M. Sancho *ibid.* **64**, 016129 (2001).

[7] A. Rocco, L. Ramirez-Piscina, and J. Casedemunt, Phys. Rev. E **65**, 056116 (2002).

[8] F. G. Bass and R. Bakanas, Europhys. Lett. **53**, 444 (2001).

[9] R. Bakanas, Nonlinearity **16**, 313 (2003).

[10] F. Schlogl, C. Escher, and R. S. Berry, Phys. Rev. A **27**, 383 (1983); J. Dziarmaga and W. Zakrzewsky, Phys. Lett. A **242**, 227 (1998); F. G. Bass and R. Bakanas, Waves Random Media **10**, 217 (2000).

[11] Jing-hui Li, Phys. Rev. E **67**, 061110 (2003); B-Q. Ai, X-J. Wang, G-T. Liu, H-Zh. Xie, D-H. Wen, W. Chen, and L-G. Liu, e-print, physics/0306163.

[12] L. Morales-Molina, N. R. Quintero, F. G. Mertens, and A. Sanchez, e-print, cond-mat/0305207.